

# CANTILEVERS

LOADING	MOMENT	SHEAR	DEFLECTION
	$M_{max} = W \left( a + \frac{b}{2} \right)$	$R_A = W$	$d_{max} = \frac{W(8a^3 + 18a^2b + 12ab^2 + 3b^3)}{24EI}$
	$M_x = \frac{Wx^2}{2a}$ $M_{max.} = \frac{Wa}{2}$	$R_A = W$	$d_C = \frac{Wa^3}{8EI}$ $d_{max.} = \frac{Wa^3}{8EI} \left( 1 + \frac{4b}{3a} \right)$
	$M_{max.} = W \left( a + \frac{b}{2} \right)$	$R_A = W$	$d_{max.} = \frac{W}{24EI} x$ $(8a^3 + 18a^2b + 12ab^2 + 3b^3 + 12a^2c + 12abc + 4b^2c)$
	$M_x = \frac{Wx^3}{3a^2}$ $M_A = \frac{Wa}{3}$	$R_A = W$	$d_C = \frac{Wq^3}{15EI}$ $d_{max.} = \frac{Wq^3}{15EI} \left( 1 + \frac{5b}{4a} \right)$

## CANTILEVERS

LOADING	MOMENT	SHEAR	DEFLECTION
	$M_x = \frac{Wx}{3} \left[ \left(\frac{x}{a}\right)^3 - \frac{3x}{a} + 2 \right]$ $M_A = \frac{2Wx}{3}$	$R_A = W$	$d_C = \frac{11Wx^3}{60EI}$ $d_{max.} = \frac{11Wx^3}{60EI} \left( 1 + \frac{15b}{11a} \right)$
	$M_{max.} = W \left( a + \frac{2b}{3} \right)$	$R_A = W$	$d_{max.} = \frac{W(20a^3 + 50a^2b + 40ab^2 + 11b^3)}{60EI}$
	$M_x = P \cdot x$ $M_{max} = P \cdot a$	$R_A = P$	$d_C = \frac{Pa^3}{3EI}$ $d_{max.} = \frac{Pa^3}{3EI} \left( 1 + \frac{3b}{2a} \right)$
	$M_{max.} = M_x = M_C$	<p style="text-align: center;">No shears</p>	$d_C = \frac{M \cdot a^2}{2EI}$ $d_{max.} = \frac{M \cdot a^2}{2EI} \left( 1 + \frac{2b}{a} \right)$
			<p><i>N.B. For anti-clockwise moments the deflection is upwards.</i></p>

## SIMPLY SUPPORTED BEAMS

LOADING

MOMENT

SHEAR

DEFLECTION

LOADING

MOMENT

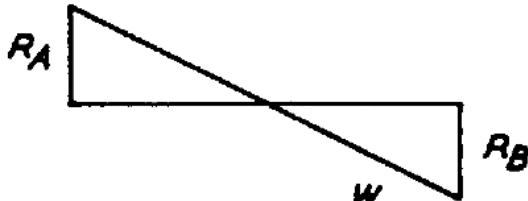
SHEAR

DEFLECTION

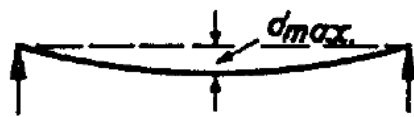


$$M_x = \frac{Wx}{2} \left(1 - \frac{x}{L}\right)$$

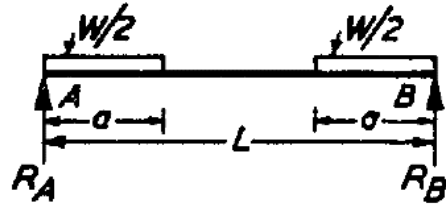
$$M_{max.} = \frac{WL}{8}$$



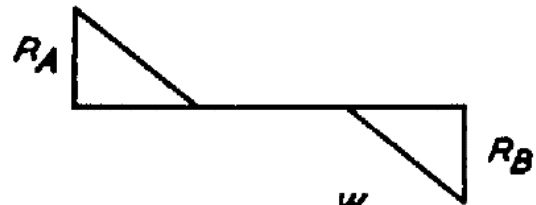
$$R_A = R_B = \frac{W}{2}$$



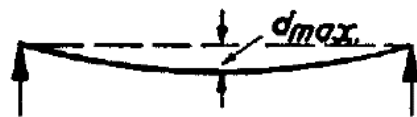
$$d_{max.} = \frac{5}{384} \cdot \frac{WL^3}{EI}$$



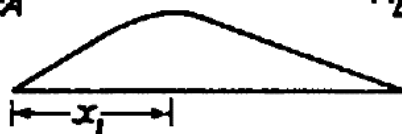
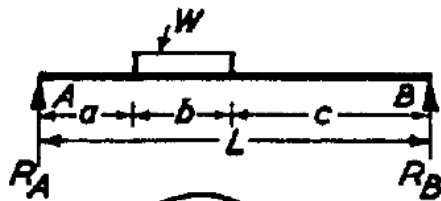
$$M_{max.} = \frac{Wa}{4}$$



$$R_A = R_B = \frac{W}{2}$$

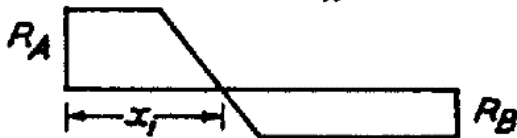


$$d_{max.} = \frac{Wa(3L^2 - 2a^2)}{96EI}$$



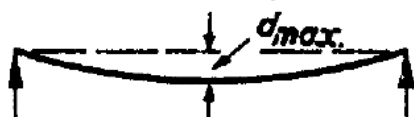
$$M_{max.} = \frac{W}{b} \left( \frac{x_1^2 - a^2}{2} \right)$$

when  $x_1 = a + \frac{Rab}{W}$



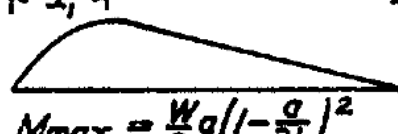
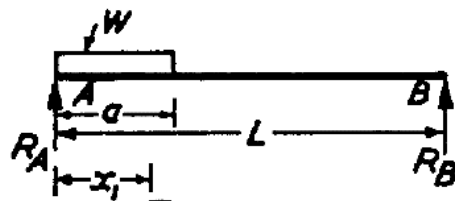
$$R_A = \frac{W}{L} \left( \frac{b}{2} + c \right)$$

$$R_B = \frac{W}{L} \left( \frac{b}{2} + a \right)$$



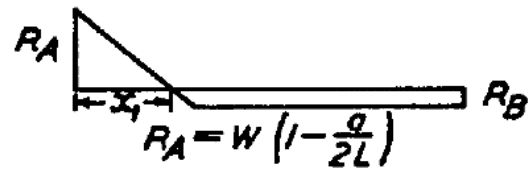
When  $a = c$

$$d_{max.} = \frac{W}{384EI} (8L^3 - 4Lb^2 + b^3)$$



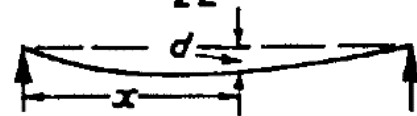
$$M_{max.} = \frac{W}{2} a \left(1 - \frac{a}{2L}\right)^2$$

when  $x_1 = a \left(1 - \frac{a}{2L}\right)$



$$R_A = W \left(1 - \frac{a}{2L}\right)$$

$$R_B = \frac{Wa}{2L}$$



When  $x \leq a$ ,

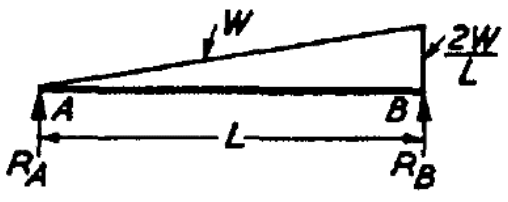

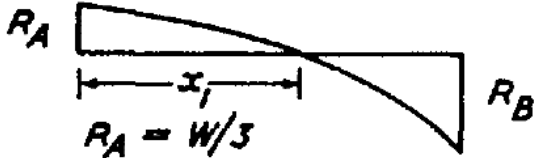
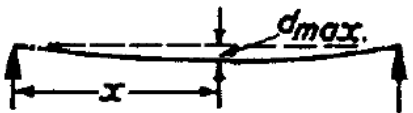


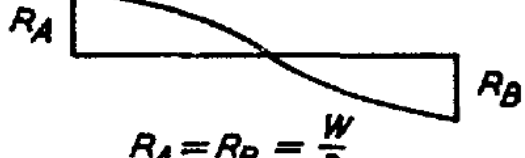
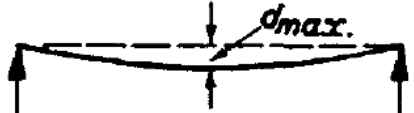
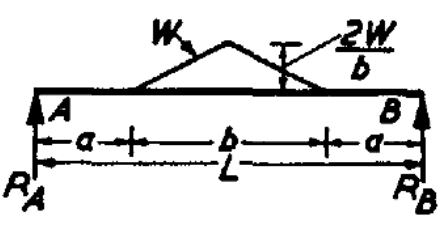

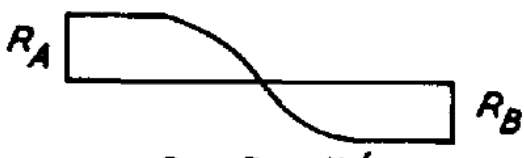
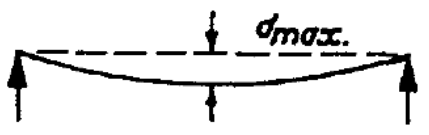
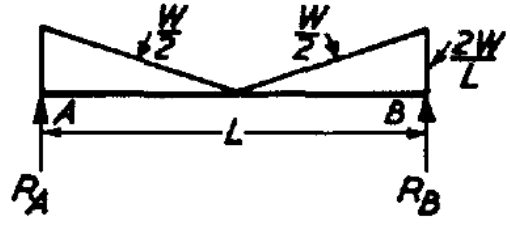


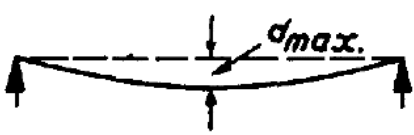
$$d = \frac{WL^2}{240EI} [m^4 - 2n(2-n)m^3 + n^2(2-n)^2m]$$

When  $x > a$ ,

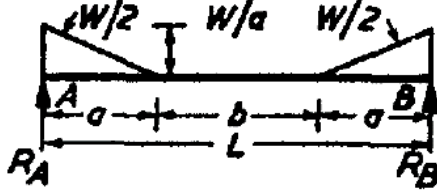

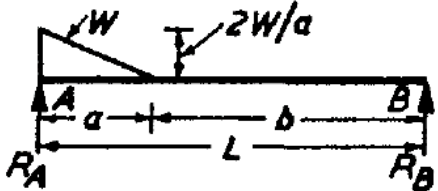
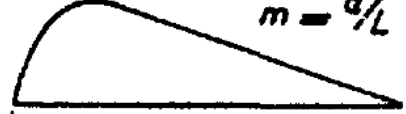

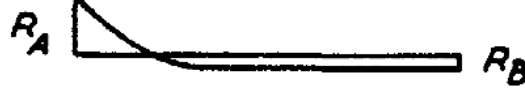
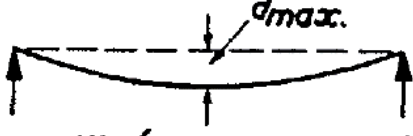
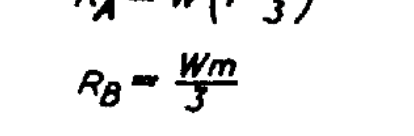
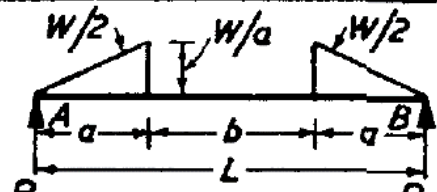

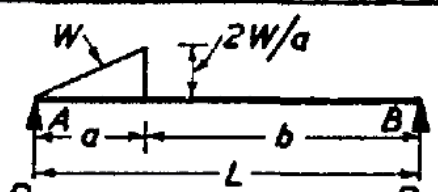

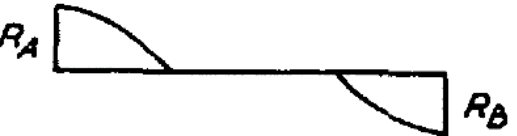

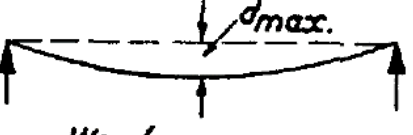
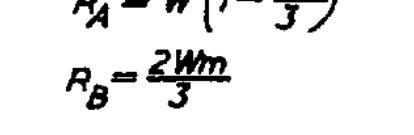
$$d = \frac{WL^2}{240EI} [n^2(2m^3 - 6m^2 + m(4+n^2) - n^2)]$$

where  $m = x/L$  and  $n = a/L$

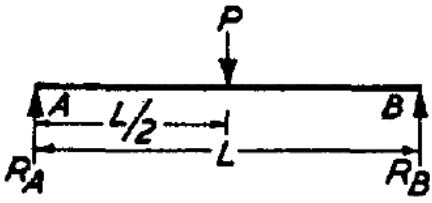

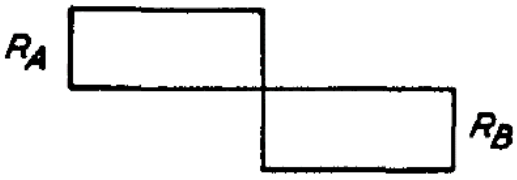
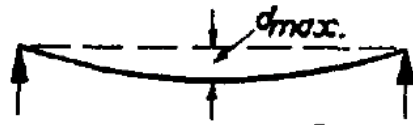
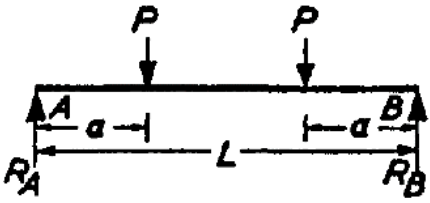

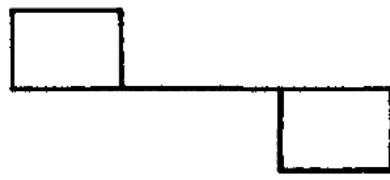
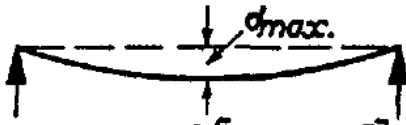
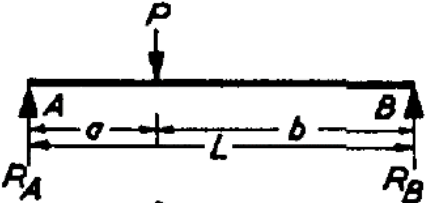
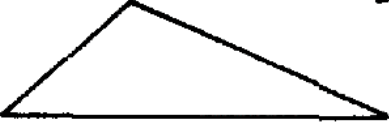
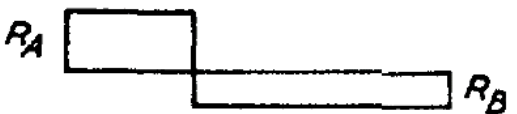
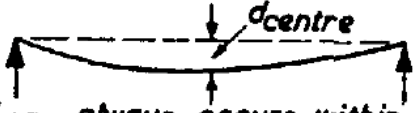
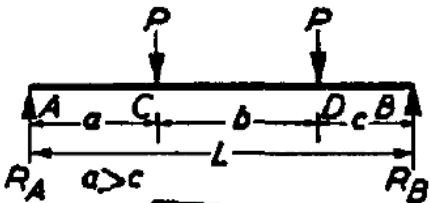

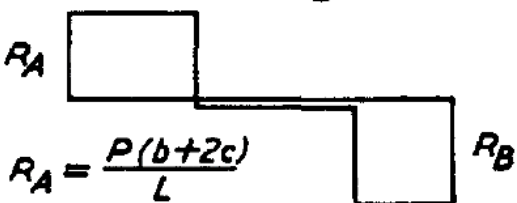
## SIMPLY SUPPORTED BEAMS

LOADING	MOMENT	SHEAR	DEFLECTION
	 $M_x = \frac{Wx}{3} \left( 1 - \frac{x^2}{L^2} \right)$ $M_{max.} = 0.128WL$ <p>when <math>x_1 = 0.5774L</math></p>	 $R_A = W/3$ $R_B = 2W/3$	 $d_{max.} = \frac{0.01304WL^3}{EI}$ <p>when <math>x = 0.5193L</math></p>
	 $M_x = Wx \left( \frac{1}{2} - \frac{2x^2}{3L^2} \right)$ $M_{max.} = WL/6$	 $R_A = R_B = \frac{W}{2}$	 $d_{max.} = \frac{WL^3}{60EI}$
	 $M_{max.} = \frac{W}{4} \left( L - \frac{b}{3} \right)$	 $R_A = R_B = W/2$	 $d_{max.} = \frac{W}{480EI} (8L^3 + 7aL^2 - 4a^2L - 4a^3)$
	 $M_x = Wx \left( \frac{1}{2} - \frac{x}{L} + \frac{2x^2}{3L^2} \right)$ $M_{max.} = WL/12$	 $R_A = R_B = \frac{W}{2}$	 $d_{max.} = \frac{3WL^3}{320EI}$

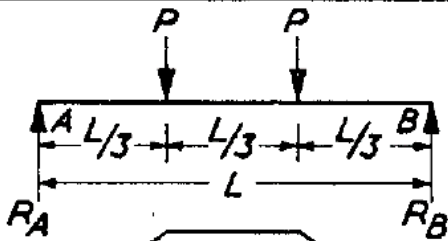
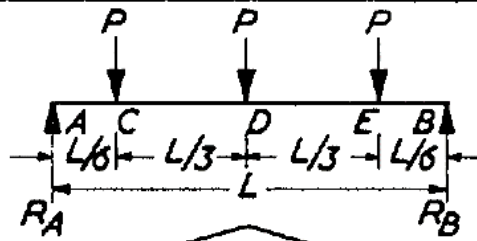
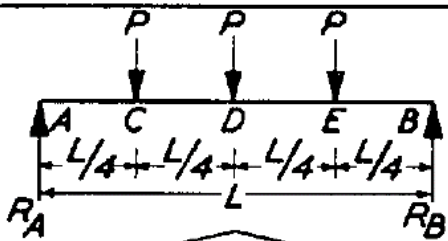
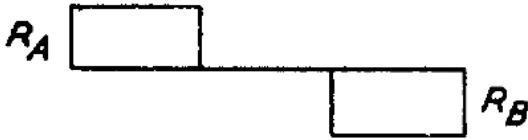
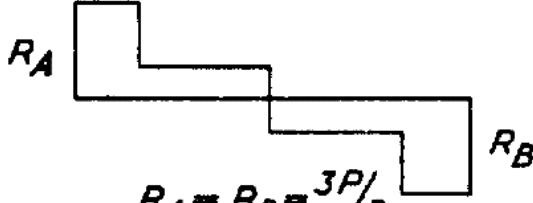
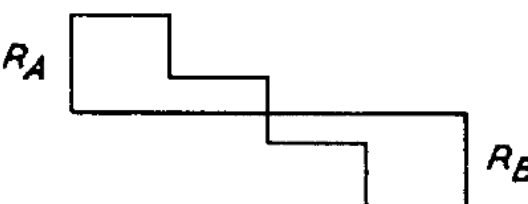
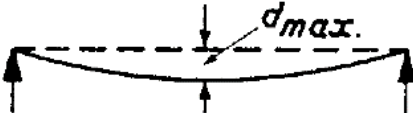
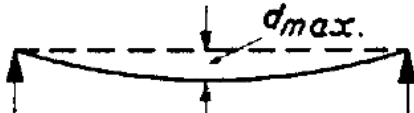
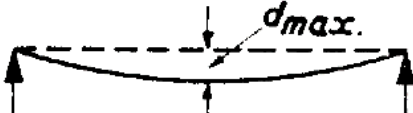
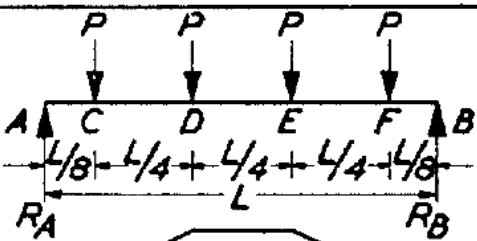
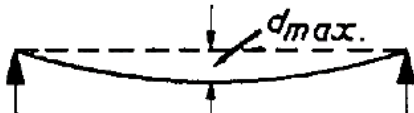
SIMPLY SUPPORTED BEAMS

LOADING	LOADING	LOADING
MOMENT	  $M_{max.} = \frac{Wa}{6}$	  $M_{max.} = \frac{Wa}{3} \left( 1 - m + \frac{2m}{3} \sqrt{\frac{m}{3}} \right)$ <p style="text-align: center;">when <math>x = a \left( 1 - \sqrt{\frac{m}{3}} \right)</math></p>
SHEAR	 $R_A = R_B = W/2$	 $R_A = W \left( 1 - \frac{m}{3} \right)$ $R_B = \frac{Wm}{3}$
DEFLECTION	 $d_{max.} = \frac{Wa}{240EI} (18a^2 + 20ab + 5b^2)$	 <hr style="width: 10%; margin: auto;"/>
LOADING	  $M_{max.} = \frac{Wa}{3}$	  $M_{max.} = \frac{2Wa}{3} \left( 1 - \frac{2m}{3} \right)^{3/2}$ <p style="text-align: center;">when <math>x = a \sqrt{1 - \frac{2m}{3}}</math></p>
SHEAR	 $R_A = R_B = W/2$	 $R_A = W \left( 1 - \frac{2m}{3} \right)$ $R_B = \frac{2Wm}{3}$
DEFLECTION	 $d_{max.} = \frac{Wa}{120EI} (16a^2 + 20ab + 5b^2)$	 <hr style="width: 10%; margin: auto;"/>

# SIMPLY SUPPORTED BEAMS

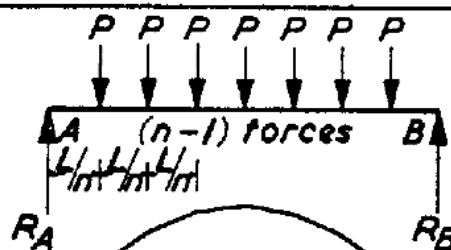
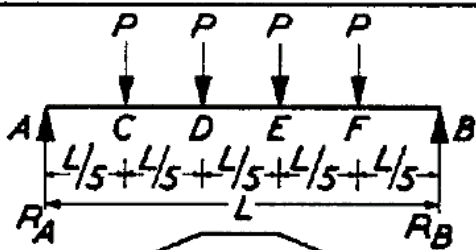
LOADING	MOMENT	SHEAR	DEFLECTION
	 $M_{max.} = \frac{PL}{4}$	 $R_A = R_B = \frac{P}{2}$	 $d_{max.} = \frac{PL^3}{48EI}$
	 $M_{max.} = Pa$	 $R_A = R_B = P$	 $d_{max.} = \frac{PL^3}{6EI} \left[ \frac{3a}{4L} - \left(\frac{a}{L}\right)^3 \right]$
	 $M_{max.} = \frac{Pab}{L}$	 $R_A = \frac{Pb}{L} \quad R_B = \frac{Pa}{L}$	 <p><i>d<sub>max.</sub> always occurs within 0.0774 L of the centre of the beam. When <math>b \geq a</math>,</i></p> $d_{centre} = \frac{PL^3}{48EI} \left[ \frac{3a}{L} - 4\left(\frac{a}{L}\right)^3 \right]$ <p><i>This value is always within 2.5% of the maximum value.</i></p>
	 $M_C = \frac{Pa(b+2c)}{L}$ $M_D = \frac{Pc(b+2a)}{L}$	 $R_A = \frac{P(b+2c)}{L}$ $R_B = \frac{P(b+2a)}{L}$	<p><i>For central deflection add the values for each P derived from the formula in the adjacent diagram.</i></p>

## SIMPLY SUPPORTED BEAMS

LOADING	LOADING	LOADING	LOADING
MOMENT	 <p style="text-align: center;"><math>M_{max.} = \frac{PL}{3}</math></p>	 <p style="text-align: center;"><math>M_C = M_E = \frac{PL}{4} \quad M_D = \frac{5PL}{12}</math></p>	 <p style="text-align: center;"><math>M_C = M_E = \frac{3PL}{8} \quad M_D = \frac{PL}{2}</math></p>
SHEAR	 <p style="text-align: center;"><math>R_A = R_B = P</math></p>	 <p style="text-align: center;"><math>R_A = R_B = \frac{3P}{2}</math></p>	 <p style="text-align: center;"><math>R_A = R_B = \frac{3P}{2}</math></p>
DEFLECTION	 <p style="text-align: center;"><math>d_{max.} = \frac{23PL^3}{648EI}</math></p>	 <p style="text-align: center;"><math>d_{max.} = \frac{53PL^3}{1296EI}</math></p>	 <p style="text-align: center;"><math>d_{max.} = \frac{19PL^3}{384EI}</math></p>
DEFLECTION	 <p style="text-align: center;"><math>M_C = M_F = \frac{PL}{4} \quad M_D = M_E = \frac{PL}{2}</math></p>	 <p style="text-align: center;"><math>d_{max.} = \frac{41PL^3}{768EI}</math></p>	

## SIMPLY SUPPORTED BEAMS

LOADING



MOMENT

$$M_C = M_F = \frac{2PL}{5} \quad M_D = M_E = \frac{3PL}{5}$$

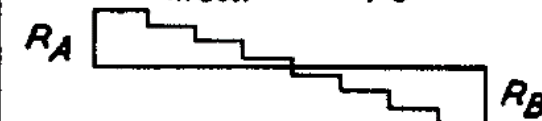
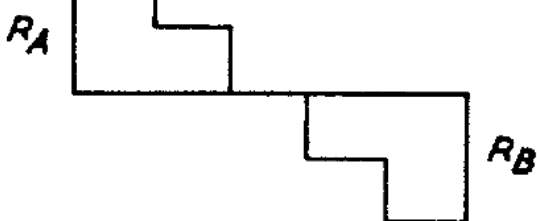
When  $n$  is odd,

$$M_{max} = \frac{(n^2 - 1) PL}{8n}$$

When  $n$  is even,

$$M_{max} = n \cdot PL / 8$$

SHEAR



$$R_A = R_B = 2P$$

$$R_A = R_B = (n-1)P/2$$

DEFLECTION

$$d_{max} = \frac{63PL^3}{1000EI}$$

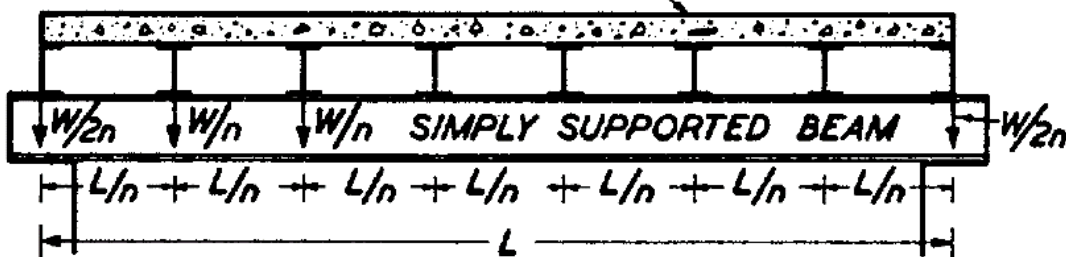
When  $n$  is odd

$$d_{max} = \frac{PL^3}{192EI} \left[ n - \frac{1}{n} \right] \left[ 3 - \frac{1}{2} \left( 1 - \frac{1}{n^2} \right) \right]$$

When  $n$  is even

$$d_{max} = \frac{PL^3}{192EI} \cdot n \left[ 3 - \frac{1}{2} \left( 1 + \frac{4}{n^2} \right) \right]$$

TOTAL LOAD =  $W$



When  $n > 10$ , consider the load uniformly distributed

The reaction at the supports =  $W/2$ , but the maximum S.F.

$$\text{at the ends of the beam} = \frac{W(n-1)}{2n} = A \cdot W$$

The value of the maximum bending moment =  $C \cdot WL$

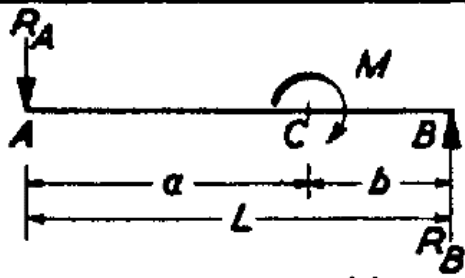
The value of the deflection at the centre of the span =  $k \cdot \frac{WL^3}{EI}$

Value of $n$	A	C	k
2	0.2500	0.1250	0.0105
3	0.3333	0.1111	0.0118
4	0.3750	0.1250	0.0124
5	0.4000	0.1200	0.0126
6	0.4167	0.1250	0.0127
7	0.4286	0.1224	0.0128
8	0.4375	0.1250	0.0128
9	0.4444	0.1236	0.0129
10	0.4500	0.1250	0.0129

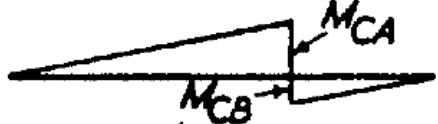


# SIMPLY SUPPORTED BEAMS

LOADING



MOMENT



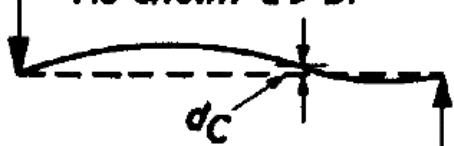
$$M_{CA} = M \cdot a/L \quad M_{CB} = M \cdot b/L$$

SHEAR



$$R_A = R_B = M/L$$

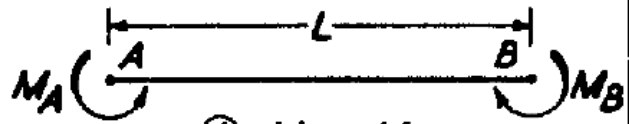
As shown  $a > b$ .



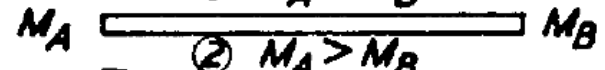
$$d_C = \frac{M \cdot ab}{3EI} \left( \frac{a}{L} - \frac{b}{L} \right)$$

For anti-clockwise moments the deflections are reversed.

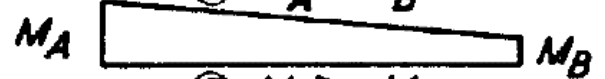
DEFLECTION



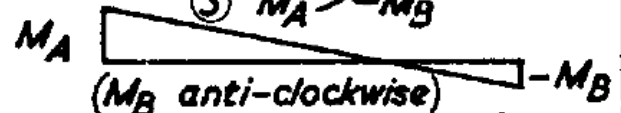
①  $M_A = M_B$



②  $M_A > M_B$



③  $M_A > -M_B$

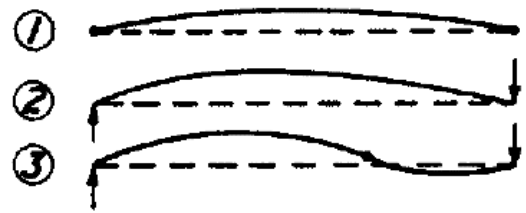


( $M_B$  anti-clockwise)

Shear diagram when  $M_A \neq M_B$



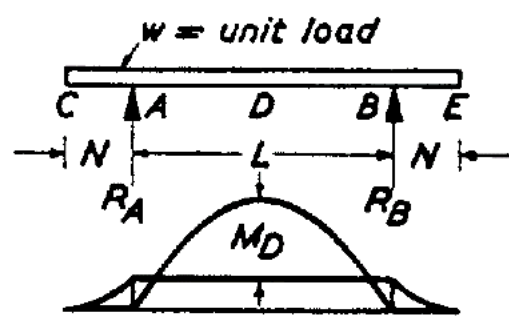
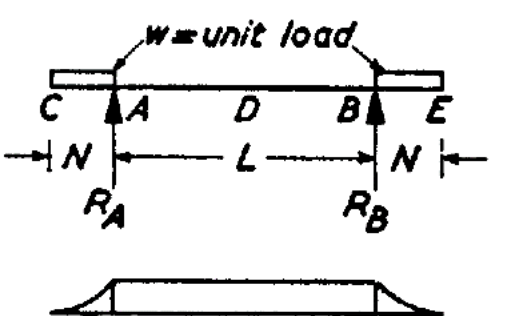
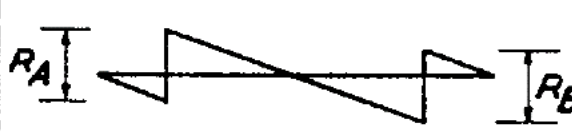
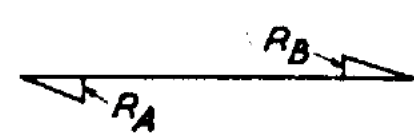
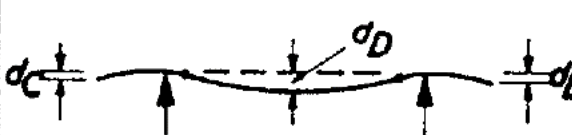

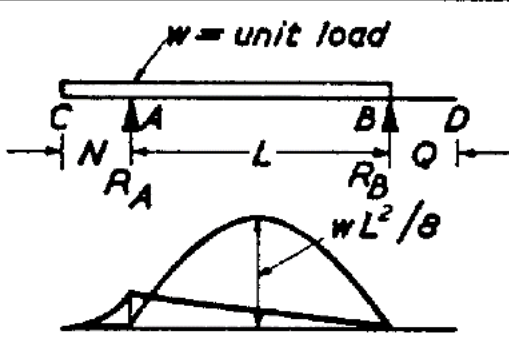
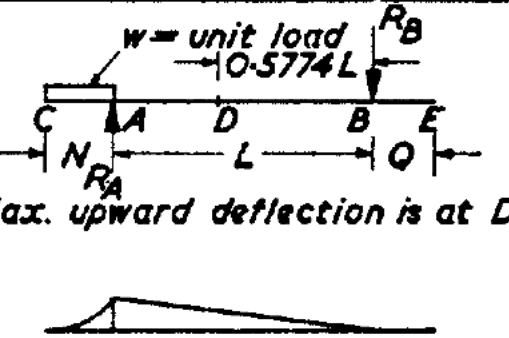
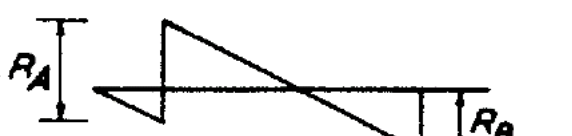
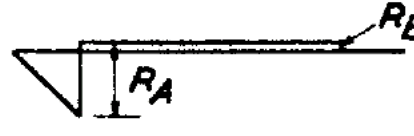
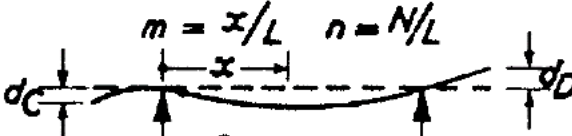
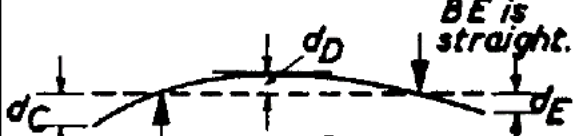
$$R_A = -R_B = \frac{M_A - M_B}{L}$$



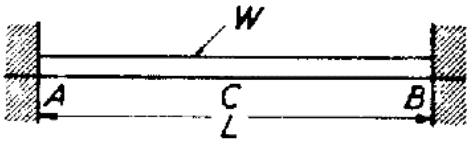
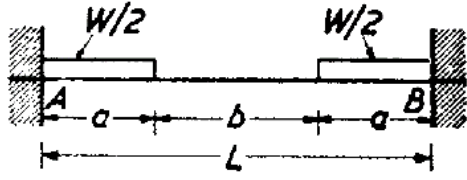

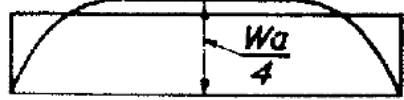
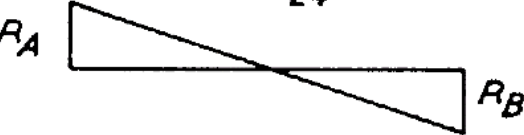
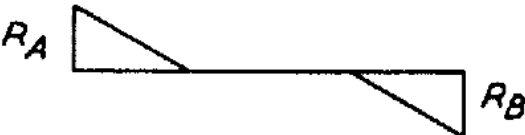
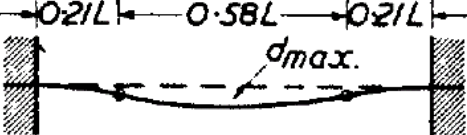
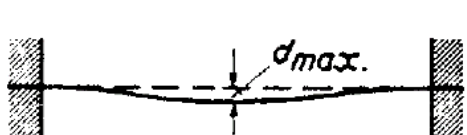
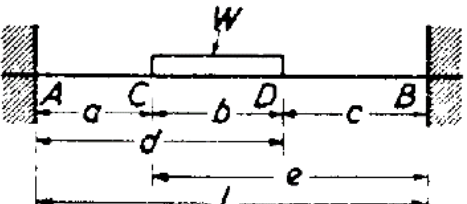
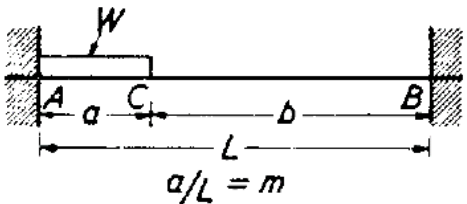
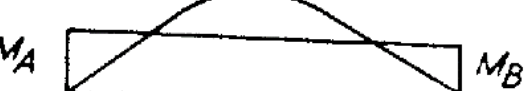

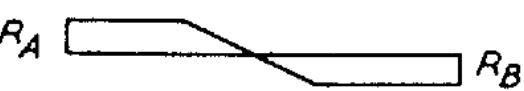
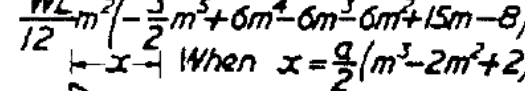
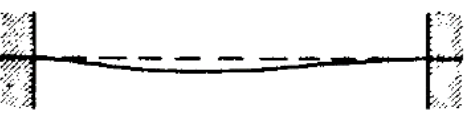
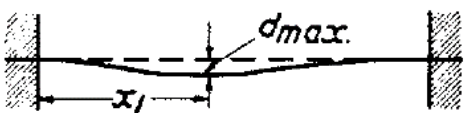
When  $M_A = M_B$ .

$$d_{max} = -\frac{ML^2}{8EI}$$

## SIMPLY SUPPORTED BEAMS

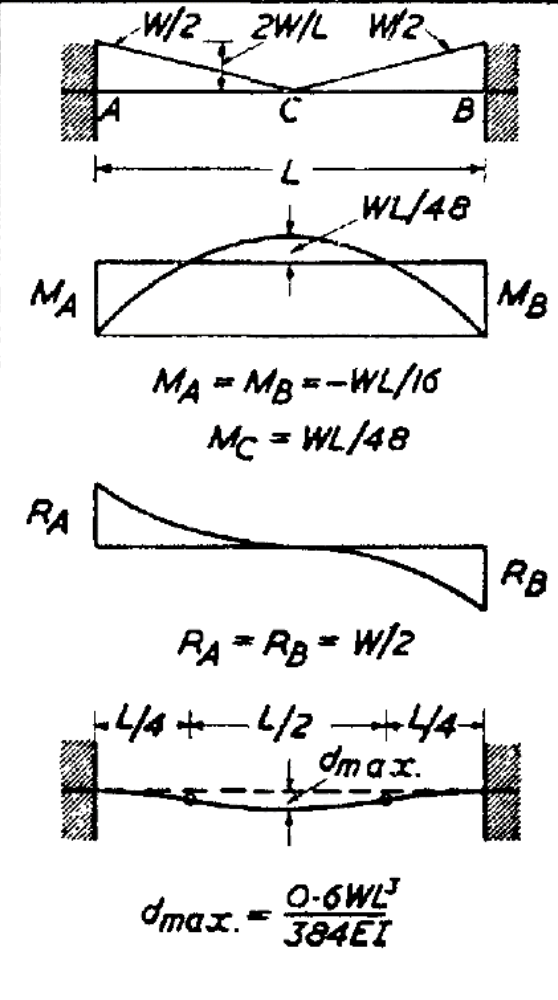
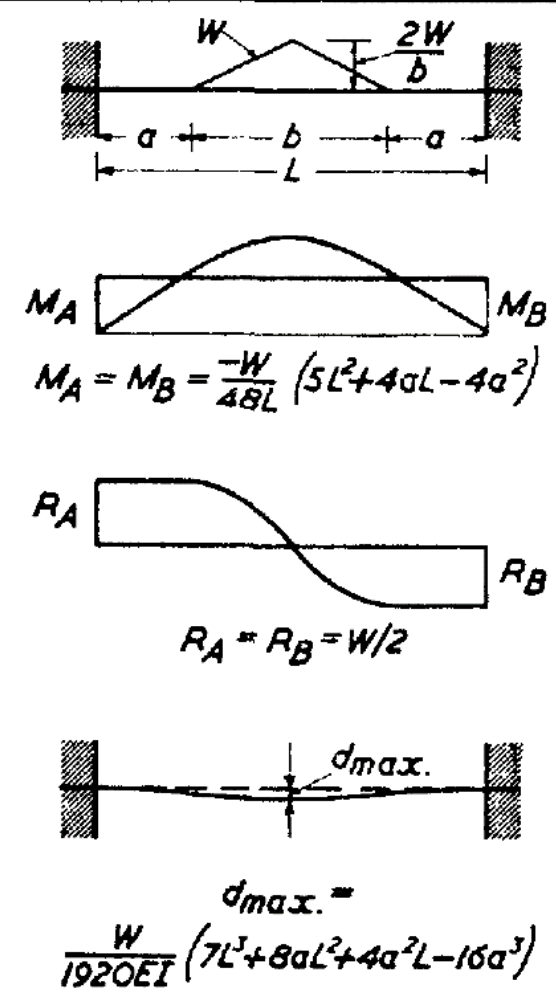
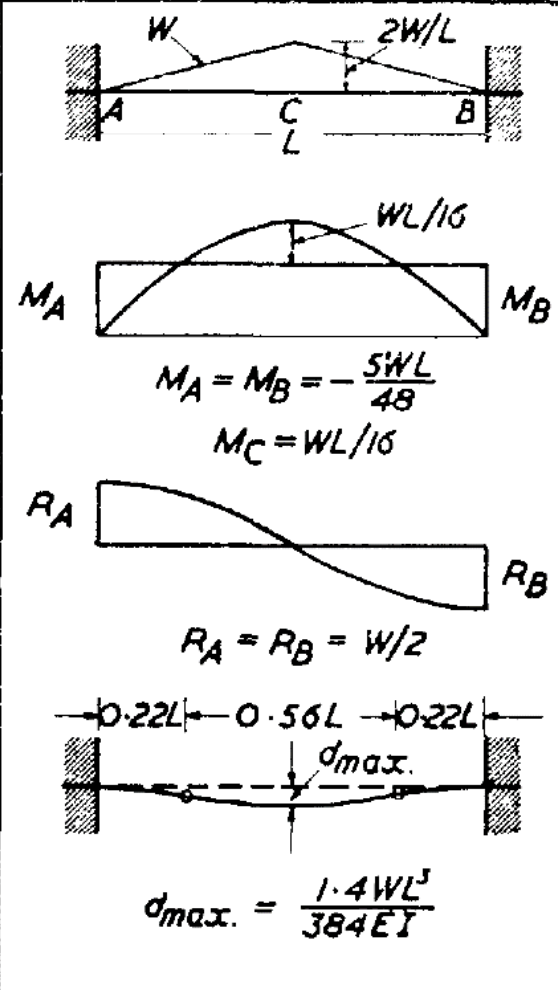
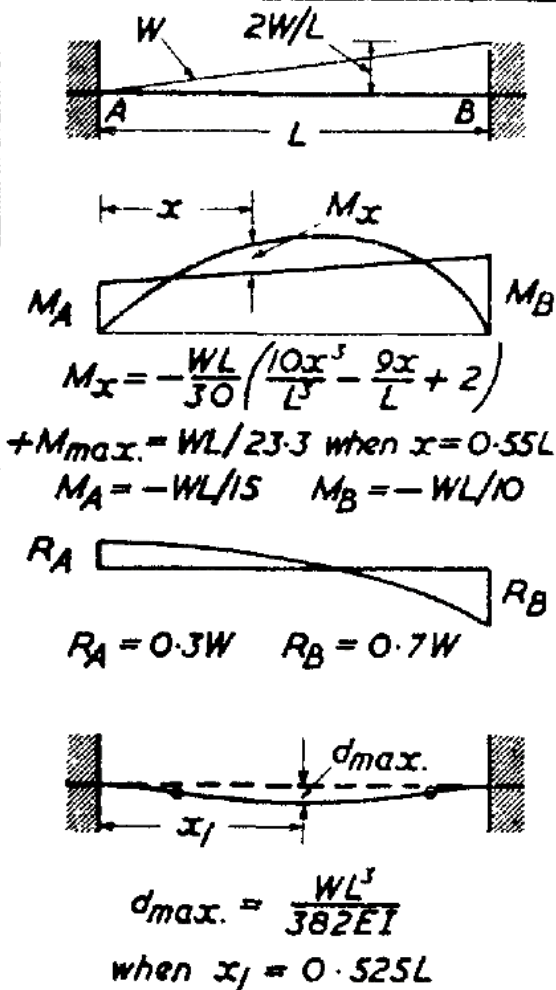
LOADING		
MOMENT	 <p style="text-align: center;"><math>w = \text{unit load}</math></p> <p style="text-align: center;"><math>M_A = M_B = -\frac{wN^2}{2}</math>    <math>M_D = \frac{wL^2}{8} + M_A</math></p>	 <p style="text-align: center;"><math>w = \text{unit load}</math></p> <p style="text-align: center;"><math>M_A = M_B = -\frac{wN^2}{2}</math></p>
SHEAR	 <p style="text-align: center;"><math>R_A = R_B = w\left(N + \frac{L}{2}\right)</math></p>	 <p style="text-align: center;"><math>R_A = R_B = wN</math></p>
DEFLECTION	 <p style="text-align: center;"><math>d_C = d_E = \frac{wL^3 N}{24EI} (3n^3 + 6n^2 - 1)</math></p> <p style="text-align: center;"><math>d_D = \frac{wL^4}{384EI} (5 - 24n^2)</math></p> <p style="text-align: center;">Where <math>n = N/L</math></p>	 <p style="text-align: center;"><math>d_C = d_E = \frac{wLN^3}{8EI} \left(2 + \frac{N}{L}\right)</math></p> <p style="text-align: center;"><math>d_D = -\frac{wL^2 N^2}{16EI}</math></p>
LOADING	 <p style="text-align: center;"><math>w = \text{unit load}</math></p> <p style="text-align: center;"><math>M_A = -\frac{wN^2}{2}</math></p>	 <p style="text-align: center;"><math>w = \text{unit load}</math></p> <p style="text-align: center;"><math>M_A = -\frac{wN^2}{2}</math></p> <p style="text-align: center;">Max. upward deflection is at D.</p>
SHEAR	 <p style="text-align: center;"><math>R_A = \frac{w(N+L)^2}{2L}</math>    <math>R_B = \frac{w(L+N)(L-N)}{2L}</math></p> <p style="text-align: center;"><math>m = x/L</math>    <math>n = N/L</math></p>	 <p style="text-align: center;"><math>R_A = \frac{wN(2L+N)}{2L}</math>    <math>R_B = \frac{wN^2}{2L}</math></p>
DEFLECTION	 <p style="text-align: center;"><math>d_C = \frac{wL^3 N}{24EI} (3n^3 + 4n^2 - 1)</math></p> <p style="text-align: center;"><math>d_x = \frac{wL^4}{24EI} [m^4 - 2m^3(1-n^2) + m(1-2n^2)]</math></p> <p style="text-align: center;"><math>d_D = -\frac{wL^3 Q}{24EI} (2n^2 - 1)</math></p>	 <p style="text-align: center;">BE is straight.</p> <p style="text-align: center;"><math>d_C = \frac{wLN^3}{24EI} \left(4 + 3\frac{N}{L}\right)</math></p> <p style="text-align: center;"><math>d_D = -\frac{0.032wL^2 N^2}{EI}</math></p> <p style="text-align: center;"><math>d_E = \frac{wLN^2 Q}{12EI}</math></p>

## BUILT-IN BEAMS

LOADING		
LOADING		
MOMENT	 $M_A = M_B = -\frac{WL}{12}$ $M_C = \frac{WL}{24}$	 $M_A = M_B = -\frac{Wa}{12L}(3L-2a)$
SHEAR	 $R_A = R_B = W/2$	 $R_A = R_B = W/2$
DEFLECTION	 $d_{max.} = \frac{WL^3}{384EI}$	 $d_{max.} = \frac{Wa^2}{48EI}(L-a)$
LOADING		
MOMENT	 $M_A = \frac{-W}{12L^2b} [e^3(4L-3e) - c^3(4L-3c)]$ $M_B = \frac{-W}{12L^2b} [d^3(4L-3d) - a^3(4L-3a)]$	 $M_A = -\frac{WL}{12} \cdot m (3m^2 - 8m + 6)$ $M_B = -\frac{WL}{12} \cdot m^2 (4 - 3m) + M_{max.} =$ $\frac{WL}{12} m^2 \left( -\frac{3}{2}m^5 + 6m^4 - 6m^3 - 6m^2 + 15m - 8 \right)$ <p style="text-align: center;"><math>x \rightarrow</math> When <math>x = \frac{a}{2}(m^3 - 2m^2 + 2)</math></p>
SHEAR	 <p>When <math>r</math> is the simple support reaction</p> $R_A = r_A + \frac{M_A - M_B}{L} \quad R_B = r_B + \frac{M_B - M_A}{L}$	 $R_A = \frac{W(m^3 - 2m^2 + 2)}{2} \quad R_B = \frac{W \cdot m^3(2 - m)}{2m}$
DEFLECTION	 <p>When <math>a = c</math>, <math>d_{max.} =</math></p> $\frac{W}{384EI} (L^3 + 2L^2a + 4La^2 - 8a^3)$	 <p>When <math>a = L/2</math> and <math>x_1 = 0.445L</math></p> $d_{max.} = \frac{WL^3}{333EI}$ $d_C = \frac{WL^3}{384EI}$

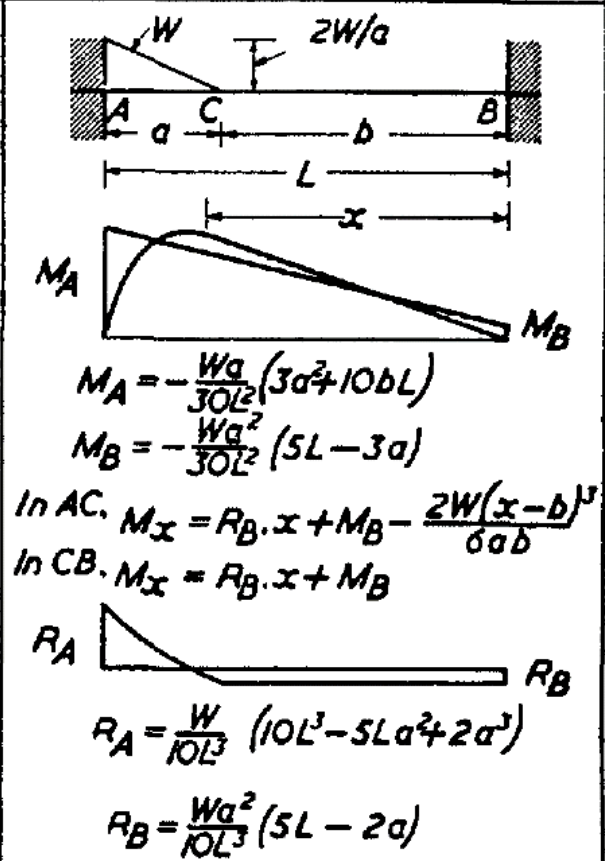
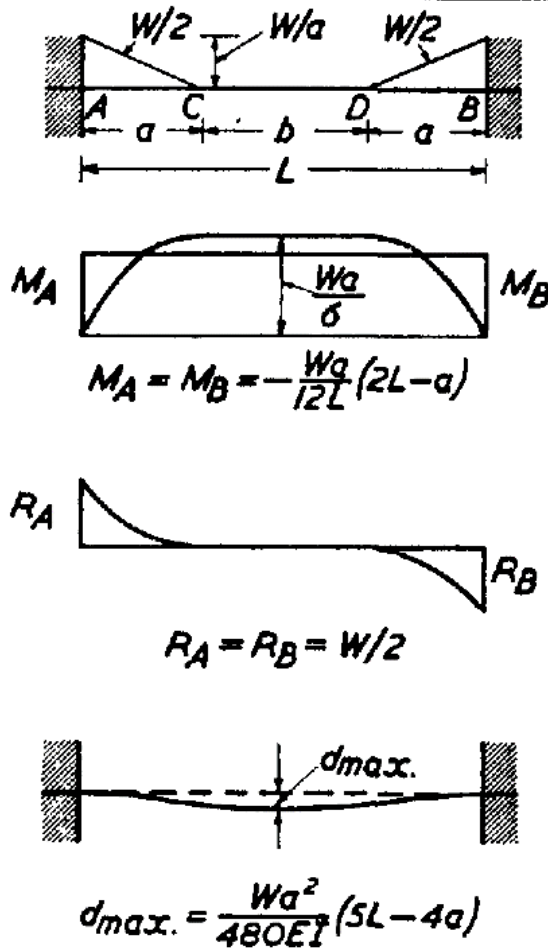
## BUILT-IN BEAMS

LOADING      MOMENT      SHEAR      DEFLECTION

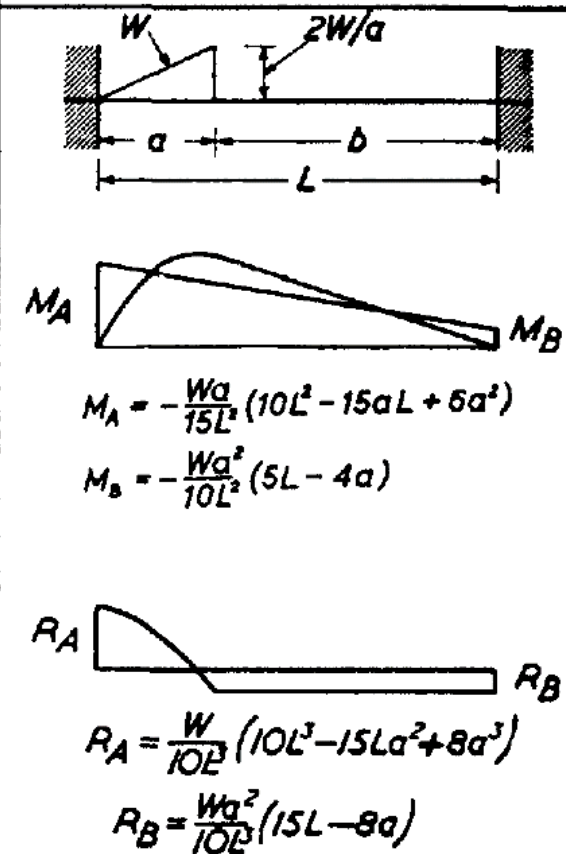
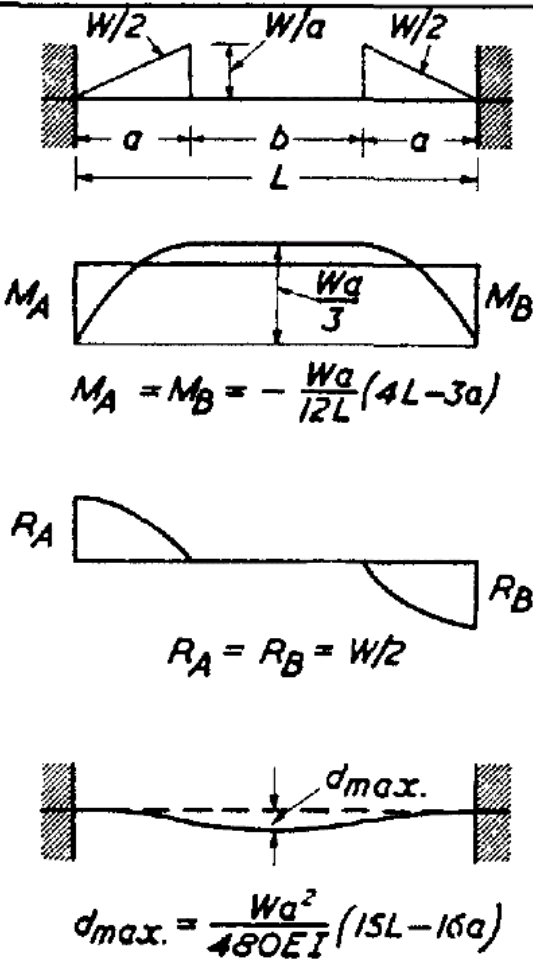


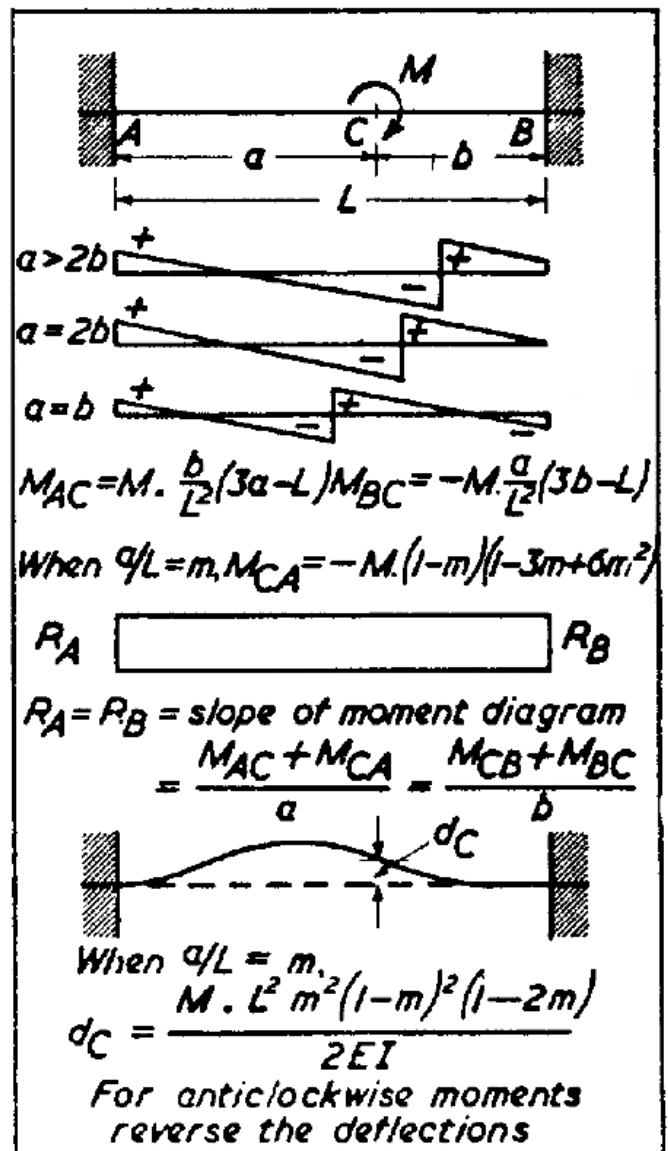
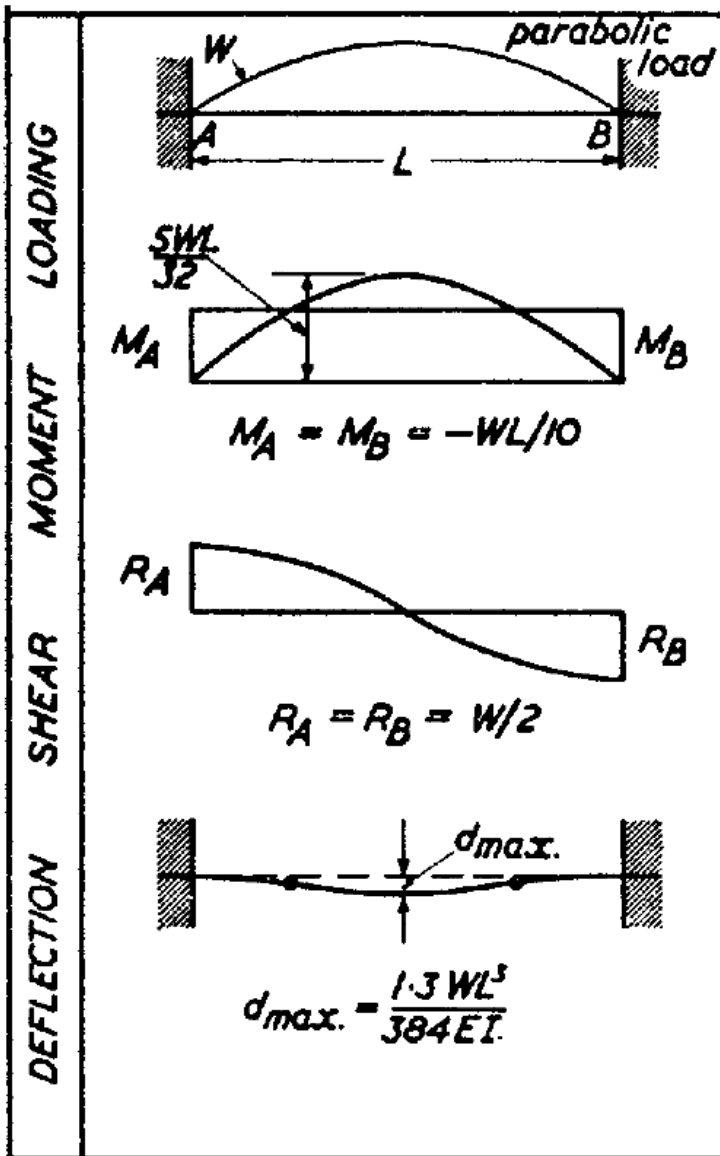
## BUILT-IN BEAMS

LOADING  
MOMENT  
SHEAR  
DEFLECTION



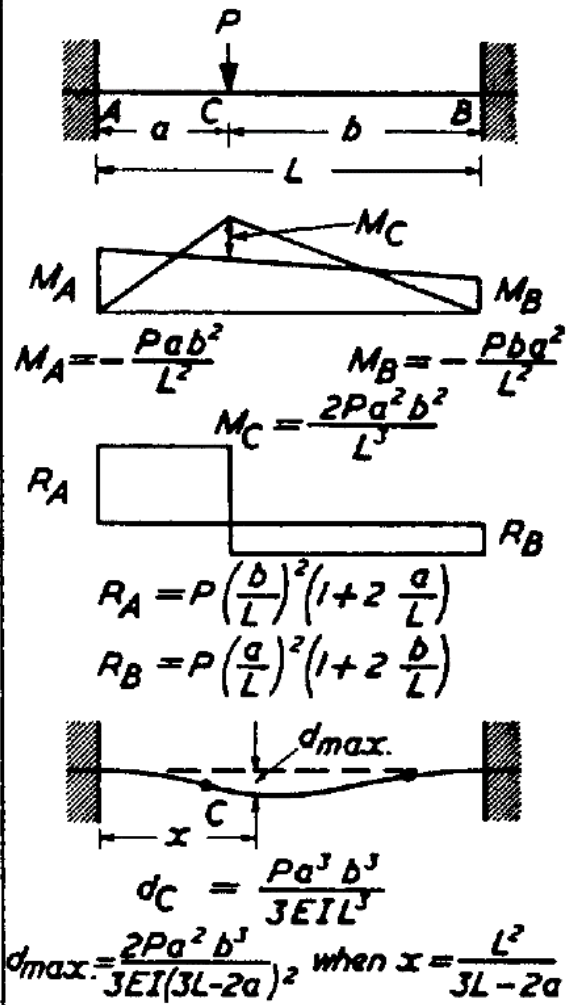
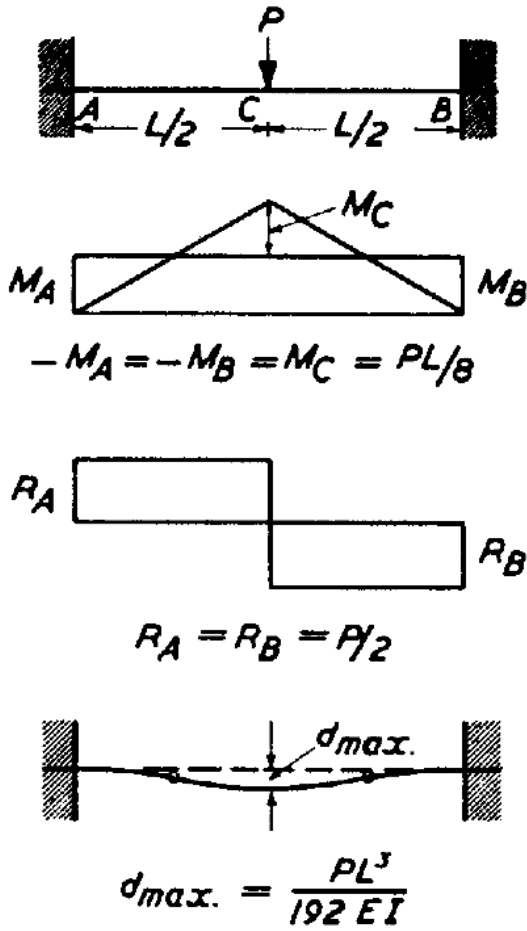
LOADING  
MOMENT  
SHEAR  
DEFLECTION



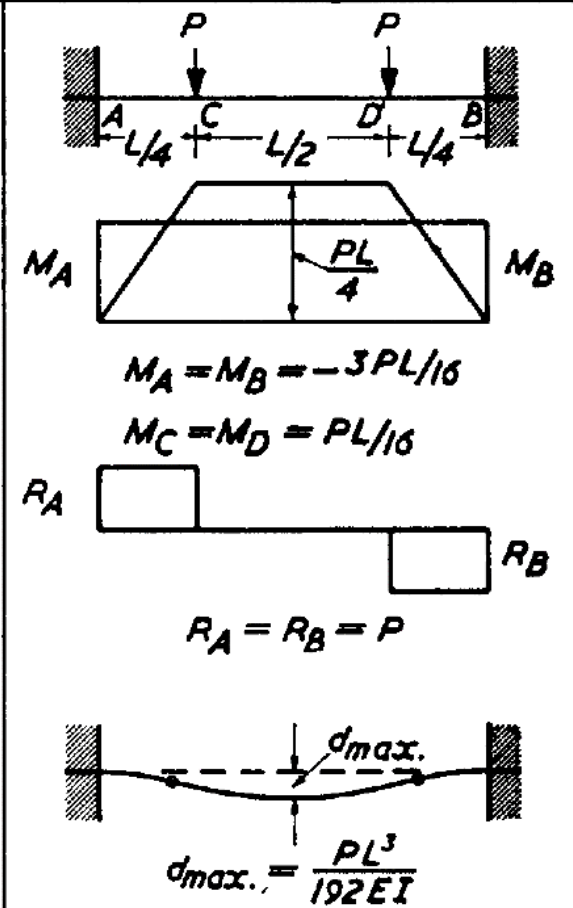
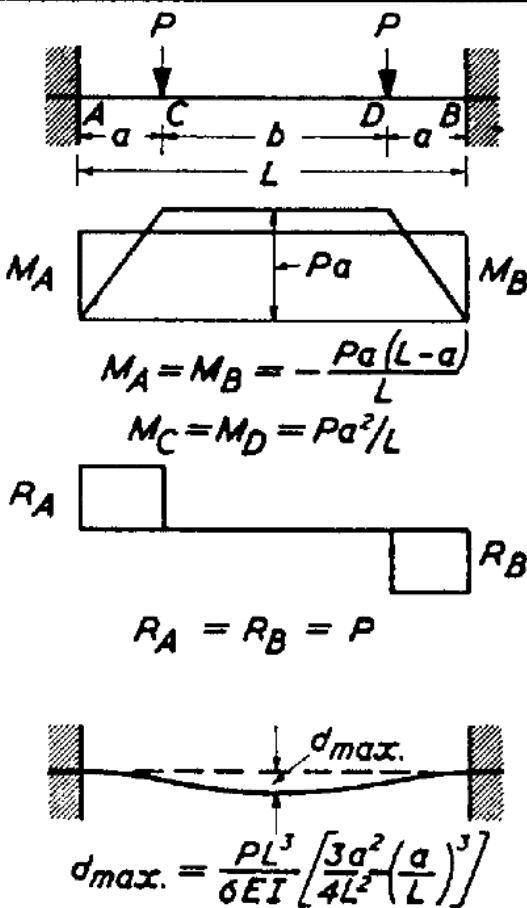


## BUILT-IN BEAMS

MOMENT · LOADING

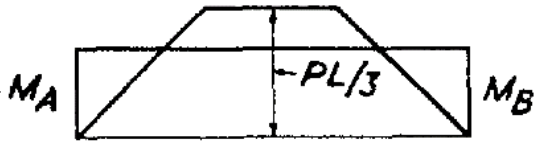
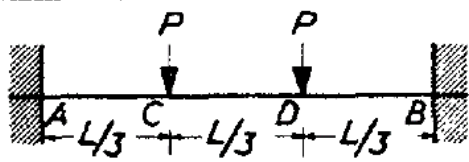


MOMENT · LOADING



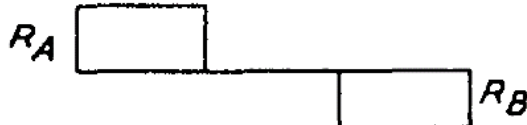
## BUILT-IN BEAMS

LOADING  
MOMENT  
SHEAR  
DEFLECTION

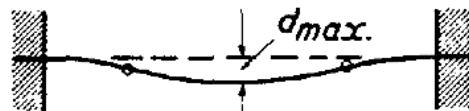


$$M_A = M_B = -2PL/9$$

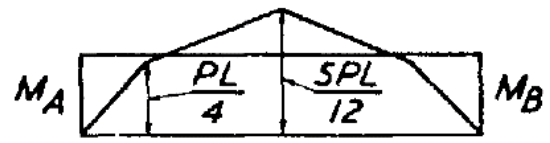
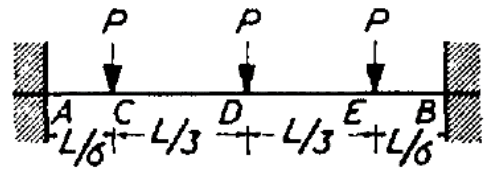
$$M_C = M_D = PL/9$$



$$R_A = R_B = P$$



$$d_{max.} = \frac{5PL^3}{648EI}$$

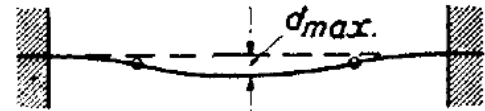


$$M_A = M_B = -19PL/72$$

$$M_D = 11PL/72$$

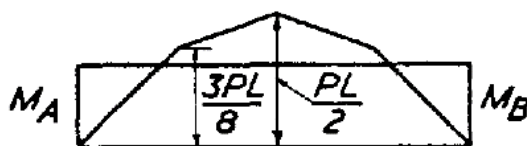
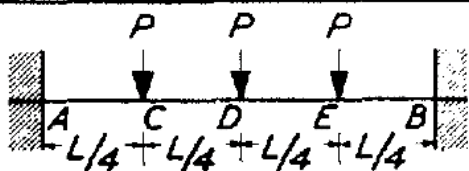


$$R_A = R_B = 3P/2$$



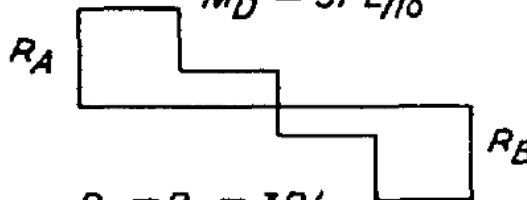
$$d_{max.} = \frac{41PL^3}{5184EI}$$

LOADING  
MOMENT  
SHEAR  
DEFLECTION

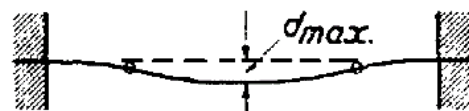


$$M_A = M_B = -5PL/16$$

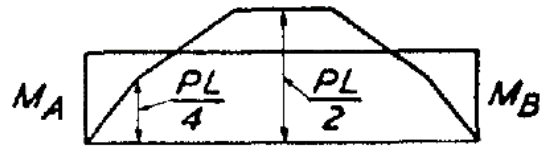
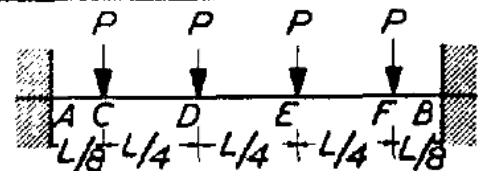
$$M_D = 3PL/16$$



$$R_A = R_B = 3P/2$$

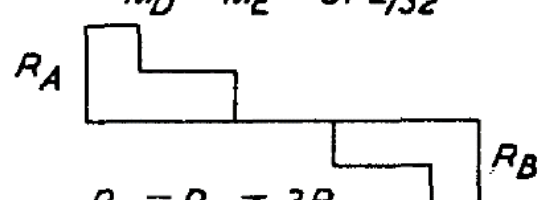


$$d_{max.} = \frac{PL^3}{96EI}$$

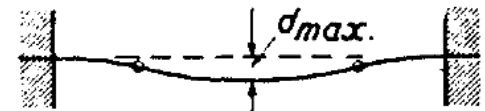


$$M_A = M_B = -11PL/32$$

$$M_D = M_E = 5PL/32$$



$$R_A = R_B = 2P$$



$$d_{max.} = \frac{PL^3}{96EI}$$